APPROACHING THEORETICAL KNOWLEDGE THROUGH VOICES AND ECHOES: A VYGOTSKIAN PERSPECTIVE

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This report deals with the ongoing construction of an innovative theoretical framework designed to organise and analyse early student approach to theoretical knowledge in compulsory education, the aim being to overcome the limits of traditional learning and constructivist hypothesis. Referring to Vygoskian analysis of the distinction between everyday and scientific concepts and the Bachtinian construct of 'voice', and drawing on previous teaching experiments, we hypothesise that the introduction in the classroom of 'voices' from the history of mathematics and science might (by means of suitable tasks) develop into a 'voices and echoes game' suitable for the mediation of some important elements of theoretical knowledge.

1. Introduction.

How to approach basic elements in modern-day scientific culture represents a serious problem in the compulsory education system. In this report we shall refer in particular to theorems, to algebraic language and to the mathematical modeling of natural and social phenomena; henceforth we shall use the term 'theoretical knowledge' to cover the above elements of mathematics. On the one hand, these are relevant for orienting and preparing students for the later study, as well as transmitting important aspects of the human cultural heritage to new generations (Boero, 1989a). On the other hand, the most common educational strategies (either traditional or not) to approach theoretical knowledge appear to be unproductive for most students, even in upper-secondary and tertiary education. In Italy as in other countries, mathematics and science theories are 'explained' by the teacher to students as from the 10th grade; the students' job is to understand them, to repeat them in verbal or written tests and to apply them in easy problem situations. The results are well known: for most students, theories are only tools for solving school exercises and do not influence their deep conceptions and ways of reasoning.

Constructivism too presents limits as regards the approach to theoretical knowledge: see Newman, Griffin & Cole (1989). We have noticed profound gaps in the aspects of mathematics mentioned at the beginning, gaps which are difficult to bridge even with the teacher's help. These are between the expressive forms of students' everyday knowledge and the expressive forms of theoretical knowledge; between the students' spontaneous way of getting knowledge through facts and theoretical deduction; and between students' intuitions and the counterintuitive content of some theories.

The ongoing research study, which is partially reported in this paper, aims to give useful elements for interpreting and overcoming the above difficulties in the approach to theoretical knowledge.

2. A Vygotskian (and Bachtinian) Perspective.

The difficulties encountered in the traditional and constructivist approaches pose a series of questions. We shall try to describe the route we have taken to reach the definitions and hypotheses presented in Section 3.

<u>What constitutes the gap between spontaneous and theoretical thinking?</u> To address this issue we have considered the distinction proposed by Vygotskij, between everyday and scientific concepts (Vygotskij, 1992, chap. VI). It is common knowledge

that this is one of the most controversial aspects of Vygotskij's work. It has often been considered outdated as it contains a systematic critique of the position taken by Piaget in the twenties, a position later revised by Piaget himself. On the other hand, the most significant examples Vygotskij uses to develop his arguments concern language and social sciences, with some generalisation to mathematics and natural sciences that are not always pertinent. In Vygotskij's school, Davydov himself has pointed out several weak and even contradictory points (Davydov, 1972). In addition, Vygotskij claims it is possible to 'teach' scientific concepts and theories to the point where they are 'internalised'; yet his hypothesis does not succeed in overcoming the learning paradox: 'How can a structure generate another structure more complex than itself?' and, more particularly, 'How does internalisation take place?' (see the discussion of Bereiter's paradox in Engeström, 1991). All the above objections have lead to underestimation of other aspects of Vygotskian analysis, such as the following: the systematic character of theoretical knowledge (versus the a-systematic nature of everyday knowledge); and the transition of scientific concepts from words to facts, versus the transition of everyday concepts from facts to word. Only recently have some researchers (e.g. John Steiner, 1995) called attention to the significance of these aspects of Vygotskian analysis. We notice that they shed light on the gap between students' everyday knowledge and theoretical knowledge, and offer a single perspective on a variety of different aspects of mathematics, such as those indicated at the beginning of this report.

Why is constructivist approach unable to bridge the gap between everyday and theoretical knowledge? On the basis of his distinction between everyday and scientific concepts, Vygotskij hypotesises that, in children's intellectual growth, their everyday knowledge has to be developed towards theoretical knowledge by establishing links with theoretical knowledge and that theoretical knowledge has to be connected with facts by establishing links with children's everyday knowledge. Yet, according to Vygotskij, the development of everyday concepts is not spontaneous: the child cannot be left alone to pursue this process because theoretical knowledge has been socially constructed in the long term of cultural history and cannot be reconstructed in the short term of the individual learning process. In short, 'exposure' to theoretical knowledge is necessary, and must be provided together with explicit links to children's knowledge.

Which aspects of theoretical knowledge are to be chosen? In our view, cultural meaning and student motivation are the most important criteria. Therefore, priority should be given to leaps forward in the cultural history of mankind, even if, for the abovementioned reasons, these are the most difficult areas for school study. The sorts of topics we are referring to include, for instance, the theory of the fall of bodies of Galilei and Newton, Mendel's probabilistic model of the transmission of hereditary traits, mathematical proof and algebraic language - all aspects with a counterintuitive character. These are 'scientific revolutions' related to historical figures from the history of science (Galilei, Newton, Mendel, Euclid, Viete). In many cases, scientific revolutions have been accomplished by overcoming epistemological obstacles (Bachelard, 1938) which were a crucial part of previous knowledge. The same obstacles are often found in individual history as well (Brousseau, 1983).

How are the leading ideas of scientific revolutions expressed? Bartolini Bussi (1995) has suggested referring to the Bachtinian construct of *'voice'* to describe some crucial elements of the turning points in scientific thinking. Bachtin's seminal work centers on literature, but some researchers in general and mathematics education have found several interesting elements therein (Bosch, 1994; Seeger, 1991; Wertsch, 1991). As far as the approach to theories is concerned, we draw on some aspects of Bachtin's work:

- the idea that human experience does not speak by itself but needs original voices that interpret it; the voices are produced in a social situation and gradually recognised by society until they become the shared way of speaking of the human experience;
- the idea that such voices act as voices belonging to real people with whom an imaginary dialogue can be conducted beyond time and space. The voices are continuosly regenerated in response to changing situations (they are not mummified voices to be listened to passively, but living tools for interpreting changing human experience).

<u>How can students be 'exposed' to the leading ideas of scientific revolutions?</u> If we transpose these ideas to the fields of science and mathematics (intended as a 'field of experience': Boero & al. 1995) we gain a useful perspective for our purposes: teachers can become mediators of 'voices' (of 'historical voices' in particular), which embody those scientific revolutions whose sense is to be conveyed to new generations. This process must take place in a social situation where the voices are renewed in accordance with changing cultural perspectives.

3. Towards a Theoretical Framework for the "Voices and echoes game".

Retrospective analysis of some teaching experiments (performed several years ago in the Genoa Group classes) confirmed the idea that scientists' voices may be exploited to approach theoretical knowledge and provided us with hints for further operational activity. As an example, let us consider the teaching experiment reported in Boero & Garuti, 1994. Students were asked to produce a brief, general statement about the relationships between heights of objects and the length of sunshadows they cast; they were asked subsequently to compare their statements with official statements of the so-called 'Thales theorem'. Analysis of the students' texts revealed an interesting phenomenon: many students had tried to rephrase their statements in order to make it resemble to the official statement, or to rephrase the official statement in order to make it to resemble their own. This was a constructive effort of a quite different nature from the production of an original statement; in fact it was an effort to 'echo' proposed 'voices'! A similar phenomenon is reported in Bartolini Bussi (1996), where the 'voice' of Piero della Francesca is exploited during a primary school perspective drawing activity.

Taking into account these experiences and the reflections summarized in the preceding section, we have undertaken the construction of a theoretical framework for a new methodological approach to theoretical knowledge. We have defined the 'voices and echoes game' and elaborated a general hypothesis concerning the effectiveness of this game in approaching theoretical knowledge (see 3.2.). Consequently, we have planned a teaching experiment, which was performed in five 8th-grade classes (see 3.3.).

Analysis of the teaching experiment allowed us to elaborate a language (see 3.4.) that we consider useful for describing, classifying and interpreting student behaviour during the 'voices and echoes' game, and which is also helpful in recognising and conveniently managing that behaviour.

We think that the research work performed so far makes it possible to plan further teaching experiments aimed at understanding better the mechanisms of individual and social cognition that allow the 'voices and echoes game' to work well; another aim would be to detect the control variables for classroom work. In summary, we consider that we have built an initial theoretical framework for a 'didactical engineering'(Artigue,1992) considered as a tool for developing research.

3.1. The 'Voices and echoes game'

Some verbal and non-verbal expressions (especially those produced by scientists of the past - but also contemporary expressions) represent in a dense and communicative way important leaps in the evolution of mathematics and science. Each of these expressions conveys a content, an organization of the discourse and the cultural horizon of the historical leap. Referring to Bachtin, we call these expressions *'voices'*.

Performing suitable tasks proposed by the teacher, the student may try to make connections between the voice and his/her own conceptions, experiences and *personal senses* (Leont'ev), and produce an *'echo'*, i.e. a link with the voice made explicit through a discourse. The 'echo' is an original idea, intended to develop our new educational methodology.

What will henceforth be called the *'voices and echoes game'* is a particular educational situation aimed at activating the production of echoes by students. To this end, specific tasks may be proposed: *'How.... might have interpreted the fact that...'*, or: *'Through what experiences ... might have supported his hypothesis';* or: *'What analogies and differences can you find between what your classmate said and what you read...',* etc. The echoes produced may become objects for classroom discussion. Some may be transformed (given appropriate stimuli and praise from the teacher) into voices which renew those introduced by the teacher and equated to the students' outlook and specific experiences.

We note that the object of the 'voices and echoes game' is not to construct a concept or an original solution to a problem, nor is it to validate a student product. Rather, the point is to compare a text (generally not produced by the student who make the comparison) with another text or with some data from everyday experience in order to detect congruences or contradictions. In this way the transition of students' thought to a theoretical level can be enhanced. Our general hypothesis on this issue is that the 'voices and echoes game' may allow the classroom's cultural horizon to embrace some elements which are difficult to construct in a constructivist approach to theoretical knowledge and difficult to mediate through a traditional approach:

- contents (especially, counter-intuitive conceptions) which are difficult to construct individually or socially;
- methods (for instance, mental experiments) far beyond the students' cultural horizon;
- kinds of organization of scientific discourse (for instance, scientific dialogue; argumentation structured into a deductive chain) which are not a natural part of students' speech.

In the case of important and counter-intuitive theories (such as Galilei's and Newton's theory of falling bodies, which was the object of our teaching experiment), we think that the transition towards the revolutionary theory should be made by overturning the contrasting theory that preceded it. Consequently, the 'voices and echoes game' should start with historical voices that give a theoretical representation of students' intuitions and interpretations. There are a number of different reason for this approach: cognitive and didactic reasons (students need to take on board epistemological obstacles - see Brousseau, 1983 and, from a different perspective, Fischbein,1994); historical and cultural reasons (important scientific changes do not happen in a cultural vacuum, but occur when new theories substitute old ones); reasons related to student transition to a theoretical dimension (a theoretical dimension may be more accessible if it initially concerns theories which resemble students' conceptions about natural phenomena or mathematical entities).

3.2. A Teaching Experiment

A teaching experiment involving the 'voices and echoes game' was performed in five 8th-grade classes, of different level, belonging to different environments, and with partially different school background. Bearing this diversity in mind, management of classroom work differed from one class to another, although the succession of voices and the tasks for the production of echoes was similar in all the classrooms.

The theories chosen for our teaching experiment concerned falling bodies. Preceding classroom experiences perfomed by the Genoa Group had shown that 8th-grade students' spontaneous knowledge about this phenomenon is limited to perceptual data, with scarce cultural elaboration. Our hypothesis was that through the 'voices and echoes game' some historic voices (Aristotle) might encapsulate student perceptions in a meaningful and precise theoretical way, while other voices (Galilei) might lead them to overturn Aristotle's theory.

Each voice was read in the classroom under the guidance of the teacher, who provided paraphrases, explanations of words, and information concerning the general cultural framework of the voice. Following each voice there were tasks that called for the production of echoes, as well as classroom discussion of some of the echoes produced .

For each class, the teaching experiment lasted from 12 to 16 hours.

Recordings of classroom discussions and individual texts were collected.

This teaching experiment produced learning results which were much better and more extensive than those usually achieved when 8th-grade students approach theoretical knowledge. The following positive aspects were common to all the classrooms (although varying in continuity and extension from class to class):

- students acquired contents, methods and ways of organizing discourse contained in the theoretical knowledge proposed to them through the voices;
- high quality scientific debate was attained at particular moments, which differed from class to class. The importance of this lies not so much in the discoveries made (in most cases they were inherent in the voices proposed by the teacher), but in the fact that ancient scientific debate was revived and related to the present cultural and expressive horizon. It can reasonably be hoped that, once constructed and experienced in the classroom, this approach may be applied to other aspects of theoretical knowledge both in later studies and in daily life. This is in line with Bachtin' s hypothesis about literature, in which the reader starts to refer the read text to her/his personal and contemporary collective experience.

In the 'voices and echoes game' situations performed in our teaching experiment, the productivity of the different phases varied from class to class, for reasons which, althought not fully clarified yet, appear to depend not only on the classroom background, but also on the peculiar dynamic evolution of the situation and the particular didactic choices of teachers. Notwithstanding the differing productivity, interesting patterns in student behaviour were observed; these allowed us to create a classification of student behaviour related to the general aims of the 'voices and echoes game' (see 3.1.)

3.3. Description and Classification of Student Behaviour

Students may produce echoes of different types (depending on tasks and personal reactions to them). First of all, we need to distinguish *individual echoes* and *collective* *echoes* (these are produced during a classroom discussion which may concern some of the individual echoes selected by the teacher).

Individual echoes can be classified as follows:

- *superficial echoes:* these are produced in an effort to perform a task requiring echo, but do not succeed in understanding the voice. These can be recognized in inappropriate use of terms and expressions deriving from the voice, contradictions, confusion between students' conceptions and those inherent in the voice, etc.
- *mechanical echoes:* precise paraphrasing of a verbal voice or the correct solution of a standard drill exercise. The student does not go beyond the level of 'mechanical echo' if she/he is incapable of exploiting the content and/or the method conveyed by the voice in order to solve a problem which differs to some extent from the situation inherent in the voice;
- *assimilation echoes:* these can be detected when the student is capable of transferring the content and/or method conveyed by the voice to other problem situations proposed by the teacher that are only partly similar to that inherent in the voice (see Matteo, Annexe). The student does not go beyond the level of the assimilation echo if his/her manner of considering natural phenomena or mathematical entities does not take the voice into account when faced with destabilising problem situations;
- *resonances:* beyond the level of assimilation, the situation of resonance is the most interesting of all. In this case the student appropriates the voice as a way of reconsidering and representing his/her experience; the distinctive sign of this situation is the ability to change linguistic register by seeking to select and investigate pertinent elements *('deepening'),* and finding examples, situations, etc. which actualize and multiply the voice appropriately *('multiplication')* (see Enzo, Annexe);
- *dissonances* (similar to resonance, but with opposition to the content and/or method conveyed by the voice).

The echoes which develop at the collective level may consist of series of individual echoes of the voice at the center of discussion *('source voice')*; these occur one after the other irrespective of classmates echoes. At the other extreme, there may be a high level of connection between successive echoes. In particular, both the examples related to the 'source voice' and the expressions and expressive registers may undergo rapid and intensive enrichment. In other words, collective echoes may reveal phenomena of multiplication and deepening, by exploiting both the 'source voices' and classmates' echoes. We call this phenomenon *'multiple echo'*.

In a 'multiple echo' situation, *'classroom voices'* can be generated: these renew the 'source voices' proposed by the teacher in terms of expression and cultural references. The multiplication and deepening phenomena, stimulated by students' examples and continuously enriched by new expressions and experiences, may make it possible not only to express the content and methodological structure of the source voice using the students' own language but also to refer these to the students' cultural horizon.

We believe that the 'multiple echo' and the production of 'classroom voices' are the conditions which allowed some meaningful experiences of true scientific debate to take place during our teaching experiment.

4. Ongoing research

The above theoretical framework remains limited to the level of description and classification of student behaviour. The available data do not allow exact interpretation of the cognitive processes involved, nor do they provide reliable indications for reproducing 'voices and echoes game' situations. Further experiments currently being planned should allow us to progress from detecting the described behaviours (through the indicators quoted in the preceding section) to interpreting them, and in particular to identifying variables involved. This research should focus on the following, interrelated questions:

- when students are engaged in tasks requiring echo production, what are the mechanisms of individual and social cognition through which they appropriate the level of theoretical organisation of discourse inherent in the voices? As we saw in the introduction, this point represents one of the main elements forming the gap between student thought and theoretical thought. Considering this point, and the importance attributed by Vygotskij (1978) to imitation, we need to pay special attention to the functions of the mechanical echo (which can be easily 'forced' through suitable tasks);
- What are the cognitive and affective mechanisms through which the historical personality 'takes part' (when his voice is introduced by the teacher in the classroom) as an interlocutor in classroom debate? The effectiveness of the 'voices and echoes game' seems to depend on this imaginary 'participation' in the game (see also Bartolini Bussi, 1996);
- What are the variables (class background, kinds of tasks, suggested sign systems, etc.) which the productive development of the 'voices and echoes game' depends on, particularly in the production of resonances and the phenomenon of 'multiple echo'? Observations made so far suggest that available or suggested sign systems strongly influence multiplication and deepening phenomena at an individual level. As to 'multiple echo', we think that familiarity with collective discussion (as the place where students carry out the social construction of knowledge) is a necessary condition but is not in itself sufficient for generating this type of echo.

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Annexe

Excerpts from one of Aristotle's *voices*, which was selected for the 'voices and echoes game': *Each body moves towards its place, if it was removed with force [...]. According to its nature, fire moves upwards, earth moves downwards [...]. The reason for he heaviness or the lightness of the other bodies (i.e. compound bodies) is the difference between the simple bodies (earth, water, air, fire) which are their components. Bodies may be light, or heavy depending on the greater or lesser quantity of this or that simple body they contain*.

Task: *If you were Aristotle, what would you tell a young student of yours in order to explain why smoke moves upwards?*

Matteo's echo: *Because smoke derives from fire and does not contain earth, it tends to move upwards, due to its affinity with fire*

Enzo's echo: *Smoke is produced by fire and fire is absolutely light, but it is also produced by wood, which is heavy but is also light, so fire prevails because wood is heaviness + lightness and fire is only lightness; consequently smoke moves upwards, but not so much as fire, because it is kept downwards by the residual part of wood.*